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A study of UMTS Turbo Codes across Space Time Spreading Channel with the case of m = 1 and m = 2

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Abstract

The study presented in this paper is that of Universal Mobile Telecommunications Systems (UMTS) Turbo Codes across Space Time Spreading channel with one transmitter antenna, one receiver (m = 1) and two transmitter antennas and one receiver (m = 2) to see to what extent the bit error rate (BER) can be improved. Using the Max-log decoding algorithm and 12 iterations it is shown in the simulation results that an extra 5 dB is achievable.

Keywords

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A study of UMTS Turbo Codes across Space Time Spreading Channel with the case of $m = 1$ and $m = 2$

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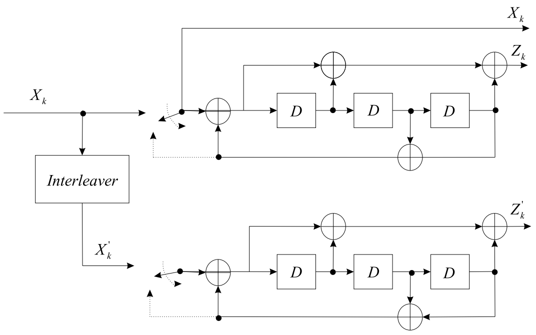


Figure 1. Block diagram of the UMTS turbo encoder

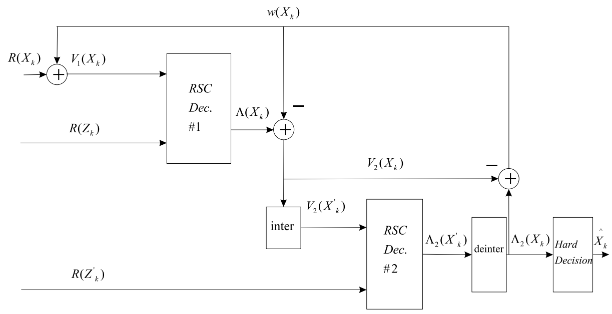


Figure 2. Block diagram of the UMTS turbo decoder

Abstract— ***The study presented in this paper is that of Universal Mobile Telecommunications Systems (UMTS) Turbo Codes across Space Time Spreading channel with one transmitter antenna, one receiver ($m = 1$) and two transmitter antennas and one receiver ($m = 2$) to see to what extent the bit error rate (BER) can be improved. Using the Max-log decoding algorithm and 12 iterations it is shown in the simulation results that an extra 5 dB is achievable.***

Key Words*-Turbo Codes, Space Time Spreading, Bit Error Rate, Transmit Diversity, Max-Log Decoding algorithm*

1. Introduction

In 1993 Berrou et al proposed in their paper "Near Shannon limit error-correcting coding and decoding: Turbo Codes" [1] a new form of error correction codes which produced excellent coding gain results near to to the Shannon theoretical limits [2]. Essentially, the Turbo Encoders are made up of two or more convolutional encoders concatenated in parallel and separated by a random interleaver, where half rate recursive systematic convolutional (RSC) encoders are often used [2]. For this presentation a set of Turbo codes called UMTS turbo codes created by Valenti et al is used [3]. Figure 1 depicts the block diagram of the UMTS turbo encoder proposed by Valenti and Sun in [3]. As with the classical turbo encoders, two recursive systematic convolutional (RSC) encoders with constraint length four are concatenated in parallel. The feed forward generator is 15 and feed back generator is 13 which is equivalent to $g = [15; 13]_{octal}$. These values are in octal. K is the input number of data bits of the turbo encoder, this value is between $40 \leq K \leq 5114$.

The two switches at the start of the process are in the up position. The data is encoded by the top encoder in its natural order while the bottom encoder encodes the bits after they are interleaved. Depending on the size of the input word, the interleaver is a matrix with 5, 10 or 20 rows with anywhere between 8 to 256 columns. The interleaver accepts the data in a row wise method, where the first data bit is in the upper-left position of the matrix. The ordering of the rows is changed by performing inter-row permutations. This process does not change the ordering of elements within each row. The data is then read in a column wise method, where the first output bit comes from the upper-left position of the transformed matrix. The overall code rate is $r = \frac{1}{3}$ because the data bits are transmitted together with the parity bits generated by the two encoders.

The order of this paper is as follows. Section 2 presents a brief description of the UMTS decoder architecture, Section 3 discusses the Max-Log decoding algorithm. Section 4 presents the STS channel and Section 5 presents results of simulation results.

2. Architecture of UMTS Turbo Codes

Figure 2 depicts the block diagram of the UMTS turbo decoder proposed by Valenti and Sun in [3]. The decoder, as with the classical turbo decoder proposed by Berrou et al in [1], is iterative. This is indicated by the feedback shown in Figure 2. Two half-iterations, one for each constituent RSC code, make up a full iteration. The extrinsic information, $w(X_k)$, is produced by the second decoder, which becomes the input of the first decoder. Before the first iteration, the extrinsic information $w(X_k)$ is set to all zeros, due to decoder number two not carrying out any tasks. The extrinsic information is updated after every iteration. Due to the two encoders having independent tails, information which is only regarded as the actual data bits is passed between decoders.

It is sufficient to add the extrinsic information $w(X_k)$ to the received systematic LLR $R(x_k)$ due to the method of deriving the branch metrics, forming a new variable $V_1(X_k)$. The input to RSC decoder number one, for $1 \le k \le K$, is the combined systematic data and the extrinsic information $V_1(X_k)$, with the received parity bits in LLR form $R(Z_k)$. The LLR $\Lambda_1(X_k)$ is the output of the first RSC decoder.

$V_2(X_k)$ is formed when $w(X_k)$ is subtracted from $\Lambda_1(X_k)$. Again, $V_2(X_k)$ holds the sum of the systematic channel LLR and the extrinsic information produced by the first decoder. The input to the second decoder is the interleaved version of $V_2(X_k)$. This input is denoted as $V_2(X_k^{'})$. The second input to decoder number two is the channel LLR corresponding to the second encoders parity bits denoted as $R(Z_k^{'})$. LLR $\Lambda_2(X_k^{'})$ is the output of the second decoder, this is de-interleaved to form $\Lambda_2(X_k)$. $V_2(X_k)$ is subtracted from the de-interleaved output of the second decoder, $\Lambda_2(X_k)$, to form the extrinsic information $w(X_k)$. This is then fed as an input to the first decoder in the next iteration.

The hard bit decision is taken after the completion of the iterations. If $\Lambda_2(X_k)$, $1 \le k \le K$, where $\hat{X}_k = 1$ when $\Lambda_2(X_k) > 0$ and $\hat{X}_k = 0$ when $\Lambda_2(X_k) \le 0$ [3].

3. The Max-Log-MAP Algorithm

This algorithm [2] converts the equations used in the MAP algorithms to a less complex format. A description of the process follows:

1. $\alpha_{k-1}(\grave{s})$ is calculated in a forward recursive manner using Equation 1.
2. $\beta_k(s)$ values are calculated in a backward recursion using Equation 2.
3. The branch transition probabilities $\gamma_s(\grave{s}, s)$ is calculated using Equation 3.

$$\alpha_k(s) = \sum_{all \grave{s}} \gamma_k(\grave{s}, s).\alpha_{k-1}(\grave{s}) \tag{1}$$

$$\beta_{k-1}(\grave{s}) = \sum_{all \grave{s}} \beta_k(s).\gamma_k(\grave{s}, s) \tag{2}$$

$$\begin{aligned} \gamma_k(\grave{s}, s) &= P(u_k)P(\underline{y_k} | \{\grave{s} \wedge s\}) \\ &= Ce^{u_k L(u_k)/2}.exp(\tfrac{E_b}{2\sigma^2} 2a \textstyle\sum_{l=1}^{n} y_{kl}x_{yl}) \\ &= Ce^{u_k L(u_k)/2}.exp(\tfrac{L_c}{2} \textstyle\sum_{l=1}^{n} y_{kl}x_{yl}) \end{aligned} \tag{3}$$

$$L(u_k|\underline{y}) = ln \frac{\sum_{(\grave{s},s) \Longrightarrow u_k=+1} \alpha_{k-1}(\grave{s})\gamma_k(\grave{s}, s)\beta_k(s)}{\sum_{(\grave{s},s) \Longrightarrow u_k=-1} \alpha_{k-1}(\grave{s})\gamma_k(\grave{s}, s)\beta_k(s)} \tag{4}$$

The simplification by the Max-Log-MAP algorithm takes place by transferring these equations into the log arithmetic domain and using the following approximation:

$$ln(\sum_i e^{x_i}) \approx \max_i(x_i) \tag{5}$$

Where $max_i(x_i)$ is the maximum value of x_i. Then $A_k(s)$, $B_k(s)$ and $\Gamma_k(\grave{s}, s)$ defined as:

$$A_k(s) = ln(\alpha_k(s)) \tag{6}$$

$$B_k(s) = ln(\beta_k(s)) \tag{7}$$

$$\Gamma_k(\grave{s}, s) = ln(\gamma_k(\grave{s}, s)) \tag{8}$$

Substituting the approximations given in Equations 6, 7 and 8 in Equation 1 yields:

$$A_k(s) \approx \max_{\grave{s}}(A_{k-1}(\grave{s}) + \Gamma_k(\grave{s}, s)) \tag{9}$$

A branch metric term $\Gamma_k(\grave{s}, s)$ is added to the previous value $A_{k-1}(\grave{s})$ to find a new value $\tilde{A}_{\grave{s}}$ for that path. Therefore, according to Equation 9, the new value of $A_k(s)$ is the maximum of the $\tilde{A}_k(s)$ values of the various paths reaching the state $S_k = s$.

The $A_k(s)$ gives the natural logarithm of the probability that the trellis is in the state $S_k = s$ at the stage *k*. Due to the approximation of Equation 5, only the maximum likelihood path through the state $S_k = s$ is considered when calculating this probability.

It then follows that the value of A_k is the Max-Log-MAP algorithm which gives the probability of the most likely path through the trellis to state $S_k = s$, rather than the probability of any path through the trellis to state $S_k = s$. Due to the approximations of this algorithm, the performance is sub-optimal compared with the MAP algorithm.

Equation 2 is converted using the approximation to form:

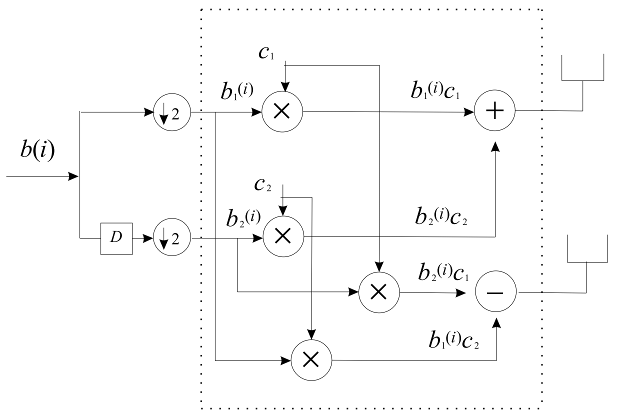
$$B_{k-1}(\grave{s}) \approx \max_{s}(B_k(s) + \Gamma_k(\grave{s}, s)) \tag{10}$$

Substituting Equations 9 and 10 into Equation 3 gives:

$$\Gamma_k(\grave{s}, s) = \hat{C} + \frac{1}{2}u_k L(u_k) + \frac{L_c}{2}\sum_{l=1}^{n} y_{kl}x_{kl} \tag{11}$$

Where $\hat{C} = lnC$ does not depend on u_k or on the transmitted codeword $\underline{x_k}$ and so can be considered a constant and omitted.

Equation 4 is then converted to form:



(a)A(2,1) STS scheme.

(b)A simple model for combination of Turbo codes and Wireless Channels.

Figure 3. STS scheme [4] and simple model of STS turbo codes

$$L(u_k|\underline{y}) \approx \begin{array}{l} max_{(\grave{s},s) \Longrightarrow u_k=+1}(A_{k-1}(\grave{s}) + \Gamma_k(\grave{s},s) + B_k(s)) \\ -max_{(\grave{s},s) \Longrightarrow u_k=-1}(A_{k-1}(\grave{s}) + \Gamma_k(\grave{s},s) + B_k(s)) \end{array} \quad (12)$$

Every transition from the trellis stage S_{k-1} to stage S_k is considered in calculating the a-posteriori LLR $L(u_k|\underline{y})$ for each bit u_k. There are two groupings which occur; $u_k = +1$ and $u_k = -1$, with each group that might have occurred if u_k equalled to the values mentioned above.

The maximum value of $(A_{k-1}(\grave{s}) + \Gamma_k(\grave{s},s) + B_k(s))$ is found for both of these groups. Then the a-posteriori LLR is calculated based on only these two 'best' transitions. There will be $2 \times 2^{k-1}$ transitions at each stage for a binary trellis. Where *K* is the constraint length of the convolutional code [2].

4. Space Time Spreading with Two Transmit Antennas

This method, called STS, spreads each user's data in a different way on each transmitter antenna. This is carried out by splitting the data into sub-streams, odd and even, $\{b_1\}$ and $\{b_2\}$ [4]. The signal transmitted on each of the antennas are presented in Equation 13 and Equation 14.

$$t_1 = (\frac{1}{\sqrt{2}})(b_1\underline{c_1} + b_2\underline{c_2}) \quad (13)$$

and

$$t_1 = (\frac{1}{\sqrt{2}})(b_2\underline{c_1} - b_1\underline{c_2}) \quad (14)$$

This is depicted in Figure 3(a).

$\underline{c_1}$ and $\underline{c_2}$ can be any set of orthogonal $2P \times 1$ unit-norm spreading sequence: $\underline{c_1}.\underline{c_2} = 0$ [4].

After de-spreading with $\underline{c_1}$ and $\underline{c_2}$, the received signals are presented in Equation 15 and Equation 16.

$$d_1 = (\frac{1}{\sqrt{2}})(h_1b_1 + h_2b_2) + \underline{c_1n} \quad (15)$$

and

$$d_2 = (\frac{1}{\sqrt{2}})(-h_2b_1 + h_1b_2) + \underline{c_2n} \quad (16)$$

The constant $\frac{1}{\sqrt{2}}$ is a normalization factor of power so a comparison can take place to a single antenna [5]. $\underline{d}$ is defined as $\underline{d} = [d_1d_2]^T$, which yields

$$\underline{d} = \frac{1}{\sqrt{2}}H\underline{d} + \underline{v} \quad (17)$$

where

$$H = \begin{bmatrix} h_1 & h_2 \\ -h_2 & h_1 \end{bmatrix} \quad (18)$$

$$\underline{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad (19)$$

$$\underline{v} = \begin{bmatrix} \underline{c_1n} \\ \underline{c_2n} \end{bmatrix} \quad (20)$$

It is then shown in [4] that by allowing $\underline{h_q}$ to denote the q^{th} column of H, you obtain Equation 21.

$$Re\{H\underline{d}\} = \frac{1}{\sqrt{2}}[|h_1|^2 + |h_2|^2]b_q + Re\{\underline{h_q}\underline{v}\} \quad (21)$$

By multiplying the de-spreading signal $\underline{d}$ by $\underline{h_1}$ and $\underline{h_2}$, the receiver can recover its odd and even symbols. This then enables the receiver to get the data into soft or hard decisions. Note that perfect knowledge of the channel coefficients is assumed and no multi-path is present. For this project, the soft decisions are needed for the Turbo Decoder to enable it to work correctly. The simulation of the STS channel system developed in [5] is used for this project. Figure 4 depicts the results for the uncoded $m = 1$ and $m = 2$ for the STS channel.

5. System Model

Figure 3(b) depicts the proposed model [6] [7] to test to what extent the turbo codes improve the bit error rate (BER) of Space Time Spreading channel with two transmit antennas and one receive antenna wireless communication. For this paper the STS channel developed in Simulink and validated in [5] [8] is used. Turbo encoder and decoder systems

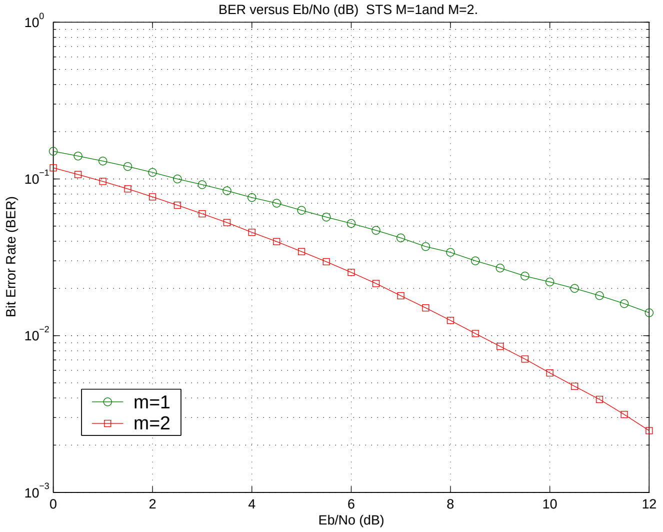


Figure 4. Plot of uncoded STS channel for $m = 1$ and $m = 2$.

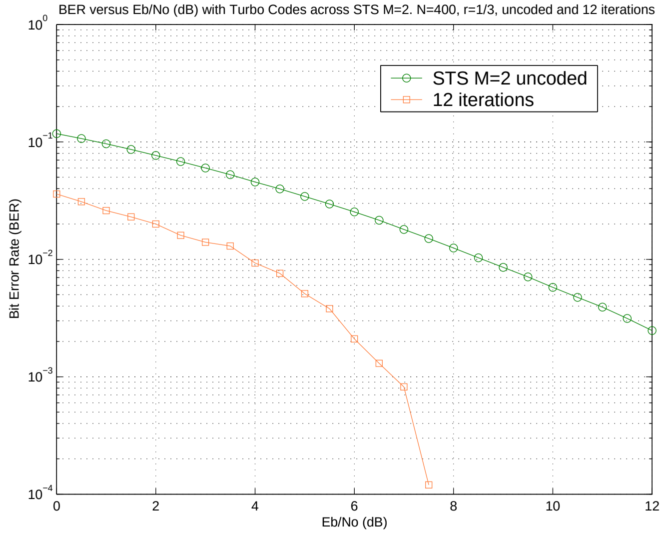


Figure 6. UMTS turbo codes across STS channel with $m = 2$, This compares uncoded BER with the 12 iterations.

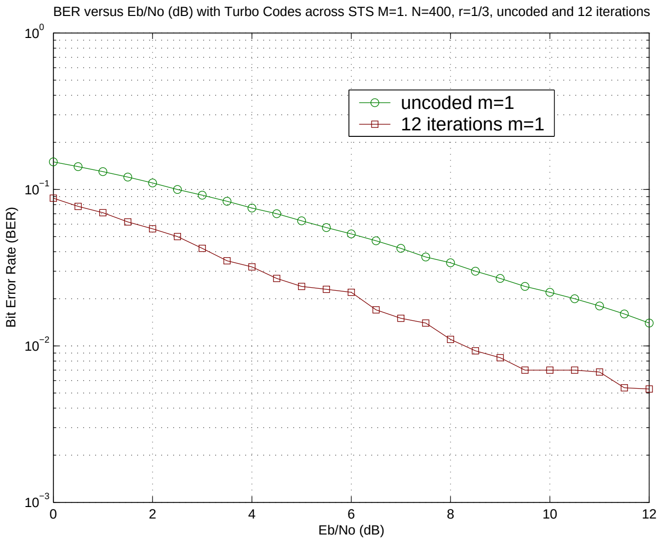


Figure 5. UMTS turbo codes across STS channel with $m = 1$, This compares uncoded BER with the 12 iterations.

are UMTS turbo codes [3] written in C and embedded into Simulink based on DeBang Lao's code from New Jersey Institute of Technology, is described in [3].

6. Results

This section shows simulation results for UMTS Turbo Codes across the STS channel for $m = 1$ and $m = 2$ using 12 iterations for the decoding. The decoding algorithm used is the Max-Log-Map described briefly is section 3. The input frame size was set to 400 and the $\frac{E_b}{N_o}$ was varied from 0dB to 12dB. This showed that an extra coding gain of up to 5 dB is achievable for the $m = 2$ case depicted in Figure 6. An extra coding gain of up to 4 dB is achievable in the $m = 1$ case depicted in Figure 5.

7. Conclusion

This paper presented a study of UMTS turbo codes across STS channel with two cases, $m = 1$ and $m = 2$, to study to what extent do turbo codes improves the bit error rate. As expected, there was improvement. An extra coding gain of 4 dB is achievable when this test is applied to the $m = 1$ case, while an extra coding gain of 5 dB is achievable when this test is applied to $m = 2$.

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